Abstract: This paper provides a novel analysis to implement a demodulation technique based on discrete Hilbert transform. Running on a DSP floating point architecture mounted in the power quality instrument PQ1000 ECAMEC fulfills the requirements of IEC61000-4-30, Class A category, over all the electrical measured parameters. Several analysis of optimal filter design and performance of signal processing are presented.

Key words: Hilbert transform, complex envelope tracking, frequency measurement, IEC 61000-4-15.

I. MATHEMATICAL FLICKER MODEL AND COMPLEX DEMODULATION

Accurate modeling of voltage flicker is a crucial task before any envelope tracking algorithm can be applied. A simplified mathematical expression of voltage flicker is expressed in (1).

\[ v(t) = A(t) \cos(2\pi ft + \alpha) \]  

where \( A(t) \) is the magnitude of voltage under flicker disturbance, \( f \), the supplied frequency and \( \alpha \), phase angle.

The time variant amplitude waveform \( A(t) \) depends on the load variability pattern. Its shape can be a step function to represent the heavy loads switching, as it can be a square or sinusoidal wave with a certain composition of frequency, or present non periodic waveform as the disturbances caused by large arc furnaces.

The goal of this study is tracking the instantaneous phasor to estimate its amplitude and phase each time a new sample of voltage waveform is acquired.

II. THE COMPLEX ENVELOPE TRACKING

The algorithm to measure complex envelope (CE) is based on the discrete Hilbert transform (HT). It is a useful mathematical tool to describe the complex envelope of a real-valued carrier modulated signals [2][3]. The CE term in this application will be named Instantaneous Phasor (IP).

The first step on signal processing is developed under the HT which provides the IP magnitude and angle. The magnitude is the voltage envelope measurement at a certain time, spectrally composed by a DC and AC components. The DC value is the sum of the amplitudes of the mains frequency and its harmonics, and the AC components is the sum of each modulation frequency amplitude (inter-harmonics) present on the point of common coupling (PCC) at the electric network.

The IP angle is the voltage angle measured at a certain time. While the main frequency component remains larger than the harmonics components, the estimated instantaneous phasor angle represents the phase of power system frequency at the PCC.

III. HILBERT TRANSFORM THEORY

The HT is defined for real signals as (2a), with its equivalent Fourier transform (FT) in (2b).

\[ x_{HT}(t) = x(t) * \frac{1}{\pi t} \]  

\[ \tilde{X}(\omega) = -j \text{sgn}(\omega) X(\omega) = \begin{cases} +j & \text{for } f > 0 \\ -j & \text{for } f < 0 \end{cases} \]  

where \( -j \text{sgn}(\omega) \) has the effect of shifting the negative frequency components of \( x(t) \) by +90º and the positive frequencies components by -90º, Fig. 1(a). The mentioned instantaneous phasor correspond to the analytical signal composed by the real signal and the Hilbert transform shifted 90º (3).

\[ IP(t) = x(t) + j x_{HT}(t) \]  

The Fourier transform of HT process is depicted on Fig. 1. The analytical signal and envelope response is shown in Fig. 1(d)

As an example the following equations (4a 4b) express how the analytical expression helps in tracking the voltage envelope and phase from the PCC.

\[ u_{rad} = A(t) \cos[\omega_0 t] \quad HT \quad u_{rad}^{HT} = A(t) \sin[\omega_0 t] \]  

\[ IP(t) = u_{rad} + j u_{rad}^{HT} \]  

\[ IP(t) = A(t) \left( \cos[\omega_0 t] + j \sin[\omega_0 t] \right) \]  

\[ IP(t) = A(t) e^{j\omega_0 t} \]  

\[ IP(t) = U_{RMS} \sqrt{2} \left( 1 + A_{FLK} \sin[\omega_{FLK} t] \right) e^{j\phi_{FLK}} \]
\[ A(t) = |IP(t)| = U_{rms} \sqrt{2} \left(1 + A_{FLK} \sin(\omega_{FLK} t)\right) \]
\[ \delta(t) = \text{angle}(IP(t)) = \arctan\left(\frac{\text{imag}(IP(t))}{\text{real}(IP(t))}\right) = \omega_{o} t \]

amplitude and instantaneous angle. Furthermore, the IP information is represented at a polar chart in Fig. 5. Far from developing a unit circle (normalized), the amplitude is governed by the flicker phenomena.

The IP components are shown in Fig. 2, at left side the cosine signal is superimposed to its Hilbert transform, a cosine shifted \(\pi/2\), i.e. a sine signal. Both signals compose the analytical expression. At right side, the analytical expression in represented on the polar graph. Each time the signal is sampled the information of amplitude and phase angle is plotted with an arrow (10 samples per cycle).

The signal sampling frequency (FS) must be constant, however this consideration do not guarantee an equidistant angular between phasors. This angle is powerful information about the voltage phase angular speed at the PCC. Angular and frequency measurement is developed at point IX of this work.

A 50Hz mains frequency signal with 10 Hz flicker frequency modulation is shown in Fig. 3. The information of IP is decomposed at Fig. 4 in its polar components: instantaneous

IV. HT – DIGITAL FILTER IMPLEMENTATION

A digital Hilbert transform technique is implemented on a floating point DSP architecture. The filter adopted is a FIR filter with odd symmetric coefficients, designed in Matlab with Parks – McClellan algorithm (using the Remez exchange algorithm and the Chebyshev approximation theory [4]). Filters designed in this way demonstrate an equiripple behavior in their frequency response.
The filter length, $N$ (number of coefficients), affects the accuracy of amplitude and phase tracking, and the speed of calculations. The longer the filter length, the minimal tracking error, but more calculations are required. The optimal length filter was carried out in a flicker-meter simulation performed on Matlab. The study included the performance analysis of demodulation at different amplitudes and frequencies of voltage modulation, considering quantized signals and filter coefficients at 24 bits.

The sampling frequency of the instantaneous phasor block is equal to 3200 samples/s, on a 24 bit ADC. The FIR HT filter length is 513 coefficient quantized at 12 bits.

V. HT – BLOCK DIAGRAM IMPLEMENTATION

In Fig. 6, the demodulation is implemented with a Hilbert discrete transform based on a FIR linear phase filter. It is recommended to choose an odd order FIR filter, because the group delay will be an integer value. After the filtering and delaying processes, the information is split.

In order to calculate the IP amplitude, $A[n]$, the information from the filtering and delaying block are arithmetically squared, added, and root squared. A trigonometric function $\text{atan2}$ is applied to the quotient of imaginary and real parts of IP to calculate instantaneous phase, $\delta[n]$.

VI. HILBERT DISCRETE FILTER DESIGN

- There are some consideration in HT discrete filter design:
  - Input signal with zero mean value: HT does not allow a DC component.
  - The voltage flicker frequency under study spreads in a range $50\text{Hz} \pm 35\text{Hz}$ or $60\text{Hz} \pm 40\text{Hz}$. The lower limits are $15\text{Hz}$ and $20\text{Hz}$, respectively.

The Parks - McClellan algorithm method design requires the filter order, transitions bands and bandwidth.

The FIR filter design must take into account the transition bands to avoid incorrect operation of demodulation at those frequencies. However the ripple of FIR filter amplitude at pass band should be limited below $\pm5\%$. In order to design the filter two independent variables must be controlled to assure the requirement, the filter order and transition band.

In order to compare different design of discrete HT FIR filters, Fig. 7, shows four cases, at a frequency rage of 0 to Nyquist (1600Hz). At first sight, the most important characteristic is the longer the order, the lower the ripple, but implies higher computational burden. Both FIR 256º and 512º order filters perform a low ripple result, below the design threshold of 0.05.

A second comparison is shown in Fig. 8 at a frequency range of $50 \pm 40\text{Hz}$, where ripple, flicker band and transition band are exposed. Again the 512º and 256º filters fulfill the design requirements.

VII. LABORATORY EQUIPMENT

The electronic and digital processing laboratory is equipped with a Fluke 6100A Electrical Power Standard. This equipment has enough uncertainty for testing Class A category of [5].

VIII. FLICKER ENVELOPE TRACKING PERFORMANCE

Fig. 9 (a) and (b) shows the demodulation processes by the discrete HT FIR filter. Two orders are performed as a comparison over a voltage flicker of $50\text{Hz}$, and $\Delta V/V~10\%$ and
8Hz of pulsation. The theoretical modulation is in dashed as a reference (Ref. Env.). The 256º order FIR gives higher level of ripple than the 512º FIR.

On Fig. 9 (b) and (c) a modulation of ∆V/V 10% is performed at 512º FIR order. Two modulation where tested at 8Hz and 4 Hz respectively. The ripple amplitude and frequency in both cases are lower than the case of 256º.

![Fig. 9 Flicker Envelope Tracking – Three comparison cases](image)

IX. FREQUENCY MEASUREMENT PERFORMANCE

The block diagram in Fig. 10 shows the derivative process of instantaneous phasor angle to estimate the power frequency involved between samples, and then is low pass filtered to minimize the noise due to derivation. This process widely satisfy the IEC 61000-4-30 [5] requirement on power frequency measurement, providing a 10s integrated value to update the sampling frequency of the data acquiring front end with a tolerance lower than ±2.5mHz in a range of 50±7.5Hz, Fig. 11.

![Fig. 10 Frequency Block Diagram](image)

![Fig. 11 Frequency error < ± 2.5mHz](image)

X. DEMODULATION DRAWBACK

The demodulation block at a flicker meter process is performed by square multiplier in [6], but this could be replaced by a Hilbert transformer, giving information of envelope amplitude and power frequency, sample per sample.

The Cigré protocol [2] proposed several tests to apply to flicker meters to assure the philosophy of [5] Class A instruments regarding to measurement comparison. The test number 5, specify the following experiment: “Sweep the frequency from 100/120 Hz to 2.0/2.4kHz (50/60Hz) at a maximum 5Hz/s slew rate.” Although isolated harmonics or isolated inter-harmonics over the 2º harmonic do not influence on flicker, spectral components at double the power frequency may change the real flicker value, due to the demodulation process. This phenomena is inherent to the demodulation method utilized.

A voltage signal composed by a power frequency voltage with flicker and a second harmonic, after the demodulation process will give flicker envelope with a no real flicker component. Its magnitude and frequency is a composition of flicker and harmonic amplitude and frequencies.

The followings equations (5a 5b) to (8) show how the square demodulation process of [6] is affected by a second harmonic. And in equation (9) and (10) how Hilbert process is influenced.

\[
A(t) = (1 + m_x \cos(\omega_x t))
\]

\[
\cos(\omega_x t) > C_1 \quad \& \quad \cos(\omega_x t) > C_2
\]

\[
u = A(t) \cos(\omega_x t) + h_x \cos(\omega_x t)
\]

\[
u = A(t)C_1 + h_x C_2
\]
From (6) the third component is extracted to analyze the addition of flicker error

\[
\mu^2 = \mu^2 + \mu^2_{\text{error}} + \mu^2_{\text{Error}}
\]

Finally (8) express the error flicker component due to the presence of a second harmonic:

\[
u_{\text{FLK}}^2 = \frac{1}{2} m_p h_2 \cos(\omega_2 - \omega_f)
\]

The flicker error component due to a Hilbert process, is similar to the square multiplier. Developing the same previous steps, we can see at (9) and (10), the mFLK error:

\[
u = A(t)C_1 + h_2 C_2
\]

The HT processes gives an error component of double the square multiplier, but its effect is reduced because the envelope calculation is the root square of the real flicker plus the error component, (10)

\[
mFLK^2_{\text{error harm}} = m_p h_2 \cos(\omega_2 - \omega_f)
\]

XI. INSTANT FLICKER PERFORMANCE

The IP amplitude is evaluated through the [6] requirements on instant flicker. Working with sinusoidal Fig. 12 and rectangular Fig. 13 modulation, the instant flicker must be unitary with a tolerance lower than ±5%.

REFERENCES